MATH5010 Linear Analysis (2020-21): Homework 5. Deadline: 1 Mar 2021

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. For each positive integer n, define a linear functional $\delta_n : c_0 \to \mathbb{K}$ by $\delta_n(x) := x(n)$ for $x \in c_0$.

   (i) Show that $\delta_n \in c_0^*$ and find $\|\delta_n\|$.

   (ii) Show that $\lim\limits_{n\to\infty} \delta_n(x) = 0$ for all $x \in c_0$ but $\delta_n \nrightarrow 0$ in c_0^*.

2. Let $X := C[-1,1]$ be the space of all real-valued continuous functions defined on $[-1,1]$. Suppose that X is endowed with the $\|\cdot\|_\infty$-norm, that is $\|f\|_\infty := \sup\{|f(x)| : x \in [-1,1]\}$. Define a linear functional φ on X by

$$\varphi(f) := \int_0^1 f(x)dx - \int_{-1}^0 f(x)dx$$

   for $f \in X$. Show that $\varphi \in X^*$ and find $\|\varphi\|$.

$* * *$ **End** $* * *$